Multi-stock WHAM and configuration for black sea bass

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Outline

- New features of multi-stock, multi-region version of WHAM
- Generalizing from univariate to multivariate abundance (multi-state)
- Ways movement, initial numbers at age, and fishing mortality are parameterized
- Calculating SPR-based biological reference points
- Projection options
- Configuring multi-stock WHAM for black sea bass base model.

New features of multi-stock, multi-region WHAM

- stock-specific abundance at age and by region
- user-defined intervals within years
- effects of environmental covariates on mortality rates by stock, region, and age
- effects of environmental covariates on recruitment by stock
- variation in movement rates by stock, region-to-region, season, age, and year
- effects of environmental covariates on movement rates by stock, region-to-region, season, and age
- mortality and movement modeled sequentially or simultaneously
- stock-specific stock-recruitment models
- priors for movement rates
- seasonal operation of fleets
- more options for initial abundance at age
- options for weighting of stock-specific SSB/R for global SPR-based reference points

Abundance transitions: Baranov

$$N_{L,t+\delta,a} = N_{t,a}S(t,\delta,a) = N_{t,a}e^{-Z_{t,a}\delta}$$

Numbers surviving from time *t to t+δ*

$$N_{C,t+\delta} = N_{t,a}H(t,\delta,a) = N_t\frac{F_{t,a}}{Z_{t,a}}\left(1 - e^{-Z_{t,a}\delta}\right)$$

Numbers captured between times *t and t+δ*

$$N_{K,t+\delta} = N_{t,a}D(t,\delta,a) = N_t\frac{M_{t,a}}{Z_{t,a}}\left(1 - e^{-Z_{t,a}\delta}\right)$$

Numbers dead due to *M* between times *t and t+δ*

Abundance transitions: vector-matrix form

$$\mathbf{N}_{t,a} = (N'_{L,t,a}, N'_{C,t,a}, N'_{K,t,a})'$$

Numbers in each state at time *t*

$$\mathbf{P}_{t,\delta,a} = \begin{bmatrix} S(t,\delta,a) & H(t,\delta,a) & D(t,\delta,a) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Probability Transition Matrix over interval δ

$$\mathbf{N}_{t+\delta,a} = \mathbf{P}'_{t,\delta,a}\mathbf{N}_{t,a}$$

Numbers in each state at time $t+\delta$

$$\mathbf{P}_{t,\delta_1+\delta_2,a} = \mathbf{P}_{t,\delta_1,a}\mathbf{P}_{t,\delta_2,a}$$

Probability Transition Matrix over interval $\delta_1 + \delta_2$

Multiple regions

$$\mathbf{P}_{t,\delta,a} = \begin{bmatrix} \mathbf{O}(t,\delta,a) & \mathbf{H}(t,\delta,a) & \mathbf{D}(t,\delta,a) \\ 0 & \mathbf{I}_H & 0 \\ 0 & 0 & \mathbf{1}_D \end{bmatrix}$$

Probability Transition Matrix over interval ***δ***

$$\mathbf{O}(t,\delta,a) = \begin{bmatrix} O_{1,1}(t,\delta,a) & \cdots & O_{1,n_R}(t,\delta,a) \\ \vdots & \ddots & \vdots \\ O_{n_R,1}(t,\delta,a) & \cdots & O_{n_R,n_R}(t,\delta,a) \end{bmatrix}$$

Probabilities of survival and moving/staying in each region over interval ***δ***

$$\mathbf{H}(t,\delta,a) = \begin{bmatrix} H_{1,1}(t,\delta,a) & \cdots & H_{1,n_F}(t,\delta,a) \\ \vdots & \ddots & \vdots \\ H_{n_R,1}(t,\delta,a) & \cdots & H_{n_R,n_F}(t,\delta,a) \end{bmatrix}$$

Probabilities of capture by each fleet over interval ***δ***

$\mathbf{D}(t,\delta,a)$ Probabilities of natural mortality over interval ***δ***

Multiple regions

- Multi-stock WHAM currently assumes each fleet operates in a single region
- Multi-stock WHAM can assume survival and movement processes are sequential or simultaneous within a seasonal interval.
  - When sequential, survival occurs over the interval and movement is assumed to happen instantly at the end of the interval

$$\mathbf{O}(t,\delta,a) = \mathbf{S}(t,\delta,a)\boldsymbol{\mu}(t,\delta,a)$$

Multiple regions

- $\mathbf{S}(t,\delta,a)$ is a diagonal matrix of proportions surviving in each region (given they start in that region):

$$\mathbf{S}(t,\delta,a) = \begin{bmatrix} e^{-Z_1(t,\delta,a)} & 0 & \cdots & 0 \\ 0 & e^{-Z_2(t,\delta,a)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & e^{-Z_R(t,\delta,a)} \end{bmatrix}$$

- $\boldsymbol{\mu}(t,\delta,a)$ is matrix of probabilities of moving from one region to another or staying (given they start in that region):

$$\boldsymbol{\mu}(t,\delta,a) = \begin{bmatrix} 1-\sum_{r'\neq 1}\mu_{1\to r'} & \mu_{1\to 2} & \cdots & \mu_{1\to R} \\ \mu_{2\to 1} & 1-\sum_{r'\neq 2}\mu_{2\to r'} & \cdots & \mu_{2\to R} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{R\to 1} & \cdots & \mu_{R\to R-1} & 1-\sum_{r'\neq R}\mu_{R\to r'} \end{bmatrix}$$

Example probability transition matrix

Northern stock, age 5, year 2021

	North	South	North_Commercial	North_Recreational	South_Commercial	South_Recreational	M
North	0.46	0.02	0.10	0.13	0.00	0.00	0.28
South	0.45	0.02	0.07	0.09	0.02	0.07	0.28
North_Commercial	0.00	0.00	1.00	0.00	0.00	0.00	0.00
North_Recreational	0.00	0.00	0.00	1.00	0.00	0.00	0.00
South_Commercial	0.00	0.00	0.00	0.00	1.00	0.00	0.00
South_Recreational	0.00	0.00	0.00	0.00	0.00	1.00	0.00
M	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Movement parameters

- Multi-stock WHAM currently has no likelihood component for tagging observations, but priors can be configured from auxiliary movement parameter estimates.
  - When prior distributions are used, the (mean) movement parameters are random effects with the mean defined by the user-specified fixed effect counterpart and standard deviation.

Initial abundance at age

The options for parameterizing initial numbers at age have been expanded in Multi-stock WHAM:

- age-specific: initial abundance at age are estimated as fixed effects.
- **equilibrium: (initial recruitment) and fully-selected F are estimated fixed effects defining equilibrium initial numbers at age.**
- iid: initial abundance at age are iid random effects with mean and variance estimated.
- ar1: initial abundance at age are are AR1 random effects with mean, variance and autocorrelation parameter estimated.

Initial abundance at age

Equilibrium assumption:

- Natural mortality and selectivity for fleet-specific fishing mortality at age are the same as those that occur during the first year of the model.
- With the assumption that each stock spawns in 1 region, there is only 1 initial recruitment parameter.
- The equilibrium calculations are essentially the same as those for SSB/R and Y/R calculations.
- For stock *s*, the equilibrium probability matrix of survival to age *a* and being in each region is

$$\mathbf{O}_{s,a}\left(\tilde{F}\right) = \begin{cases} \prod_{i=0}^{a-1} \mathbf{O}_{s,i}\left(\tilde{F}\right) & 1 \leq a < A \\ \left[\prod_{i=0}^{a-1} \mathbf{O}_{s,i}\left(\tilde{F}\right)\right] \mathbf{O}_{s,+}\left(\tilde{F}\right) & a = A \end{cases}$$

$$\mathbf{O}_0 = \mathbf{I}$$

$$\mathbf{O}_+ = \left(\mathbf{I} - \mathbf{O}_A\right)^{-1}$$

$$N_{1,r,a} = N_{1,r} \widetilde{\mathbf{O}}_a(s_r, r)$$

Fishing mortality rates

- Fishing mortality is treated the same as the standard WHAM package
- F at age for each fleet is estimated as the product of fully selected F and selectivity at age
- A fully-selected total F is also reported as the maximum of the total F at age summed across fleets

$$F_{\text{total},a,y} = \sum_{i=1}^{n_f} F_{f,a,y}$$

$$F_{\text{total},y} = \arg\max_{a} F_{\text{total},a,y}$$

- But in the multi-stock version these fleets may occur in different regions
  - The magnitude of the total F increases with more regions (that have fishing)
- There are different ways to average across fleets/regions, but the best way is debatable
- **Most important is that the representation of F in the model is consistent with that in reference points and projections**

SSB and Yield per Recruit

- The matrix of equilibrium spawning stock biomass per recruit for stock *s*, in region *r'* (columns) given recruiting in region *r* (rows) as a function of a fully-selected *F* is defined as

$$\widetilde{\mathbf{O}}_s\left(\widetilde{F}\right) = \sum_{a=1}^{A} \mathbf{O}_{s,a}\left(\widetilde{F}\right) \mathbf{O}_{s,a}\left(\widetilde{F}, \delta_s\right) \mathrm{diag}\left(\mathbf{f}_{s,a}\right)$$

- $\mathbf{O}_{s,a}(\widetilde{F}, \delta_s)$ is the matrix of probabilities of surviving and occurring in region *r'* at age *a* given starting in region *r* at age a+delta
- $\mathbf{f}_{s,a}$ is the vector of fecundities (product of weight and maturity) at age *a* in each region
- The matrix of equilibrium yield per recruit in each fleet (columns) given recruiting in a given region (rows) is calculated as

$$\widetilde{\mathbf{Y}}_s\left(\widetilde{F}\right) = \sum_{a=1}^{A} \mathbf{O}_{s,a}\left(\widetilde{F}\right) \mathbf{H}_{s,a}\left(\widetilde{F}\right) \mathrm{diag}\left(\mathbf{c}_{s,a}\right)$$

- $\mathbf{H}_{s,a}$ is the matrix of probabilities of capture in each fleet given starting in region *r*
- $\mathbf{c}_{s,a}$ is the vector of catch weight at age for each fleet

Combining SSB per Recruit across stocks

- To find F40, we use a weighted average of stock-specific components of spawning biomass per recruit as a function of fully-selected F:

$$\widetilde{S}\left(\widetilde{F}\right) = \sum_{i=1}^{n_S} \omega_i \widetilde{\mathbf{O}}_i \left(\widetilde{F}, r = r(s_i), r' = r(s_i)\right)$$

- weights can be user-specified or as a function of average recruitment for the different stocks:

$$\omega_i = \frac{\overline{R}_i}{\sum_{i=1}^{n_S} \overline{R}_i}$$

- F40 is the value of fully selected F such that:

$$\widetilde{S}\left(\widetilde{F}^*\right) = 0.4\widetilde{S}\left(\widetilde{F} = 0\right)$$

- Same Newton methods as the standard WHAM package are used to solve for F40 internally

SSB and Yield at F40

- An average recruitment is multiplied with SSB/R and Y/R for stock *i*

$$\widetilde{\mathrm{SSB}}_i = \overline{R}_i \widetilde{\mathbf{O}}_i \left(\widetilde{F}, r = r(s_i), r' = r(s_i)\right)$$

- For yield from stock *i* in fleet *f*

$$\widetilde{Y}_{i,f} = \overline{R}_i \widetilde{\mathbf{Y}}_i \left(\widetilde{F}, r = r(s_i), f\right)$$

- The total yield for fleet *f* is just the sum across stocks

$$\widetilde{Y}_f = \sum_{i=1}^{n_S} \widetilde{Y}_{i,f}$$

- User specifies the years to include for annual recruitment

BRPs example

- All inputs to SSB/R and YPR/R are averaged over last five years for BRPs under prevailing conditions
- FAA is averaged by fleet

	1	2	3	4	5	6	7	8+
North_Commercial	0.01	0.04	0.09	0.17	0.17	0.17	0.17	0.17
North_Recreational	0.01	0.09	0.10	0.15	0.26	0.40	0.56	0.56
South_Commercial	0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
South_Recreational	0.03	0.08	0.15	0.21	0.25	0.26	0.26	0.26

BRPs example

- Average fishing mortality at age and fleet is divided by the maximum of the total average FAA:

1	2	3	4	5	6	7	8+
0.05	0.23	0.4	0.61	0.76	0.91	1.07	1.07

- FAA/1.07 is selectivity at age and fleet:

	1	2	3	4	5	6	7	8+
North_Commercial	0.01	0.04	0.08	0.16	0.16	0.16	0.16	0.16
North_Recreational	0.01	0.08	0.09	0.14	0.25	0.38	0.52	0.52
South_Commercial	0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
South_Recreational	0.03	0.07	0.14	0.20	0.23	0.24	0.25	0.25

BRPs example

- Weight at age for SSB and fleets are averaged over last 5 years.
- SSB:

	1	2	3	4	5	6	7	8+
BSB_North	0.11	0.20	0.37	0.54	0.75	0.99	1.19	1.53
BSB_South	0.09	0.19	0.35	0.48	0.63	0.81	0.93	1.50

- Fleets:

	1	2	3	4	5	6	7	8+
North_Commercial	0.07	0.17	0.37	0.52	0.71	0.91	1.11	1.46
North_Recreational	0.11	0.20	0.37	0.54	0.75	0.99	1.19	1.53
South_Commercial	0.10	0.16	0.35	0.48	0.66	0.85	1.05	1.36
South_Recreational	0.09	0.19	0.35	0.48	0.63	0.81	0.93	1.50

BRPs example

- maturity, M, and movement also averaged over last 5 years.
- maturity:

	1	2	3	4	5	6	7	8+
BSB_North	0	0.48	0.98	1.00	1.00	1	1	1
BSB_South	0	0.34	0.82	0.98	0.97	1	1	1

- M:

	1	2	3	4	5	6	7	8+
BSB_North	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
BSB_South	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

- movement (Northern component):

	1	2	3	4	5	6	7	8+
North to South	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
South to North	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33

BRPs example

- F40 = 1.03. Total F40 at age (across all fleets):

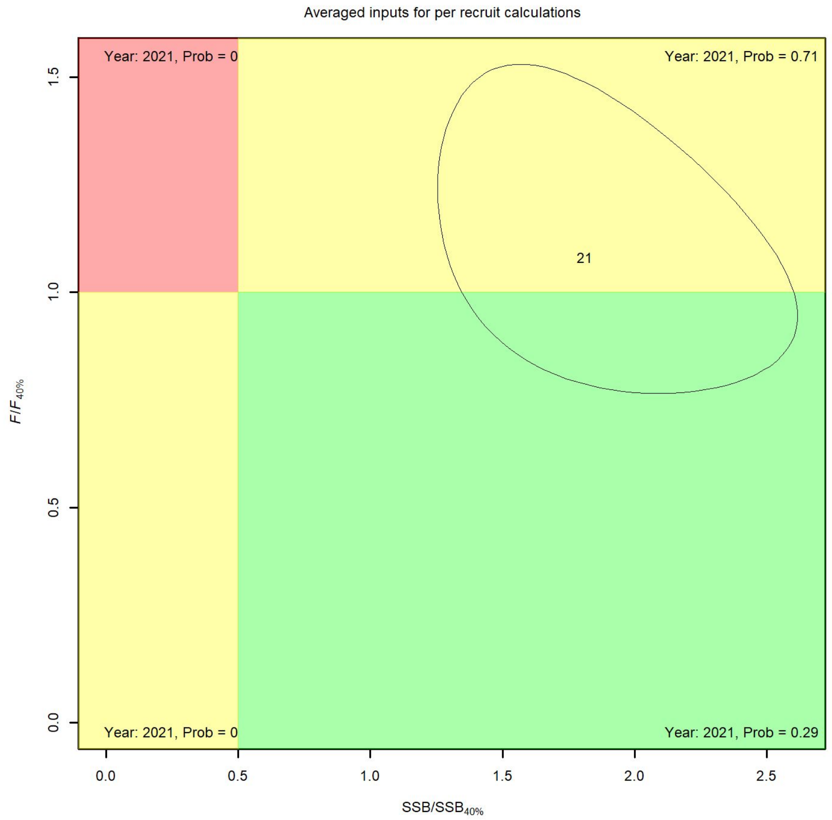
1	2	3	4	5	6	7	8+
0.05	0.22	0.39	0.59	0.73	0.88	1.03	1.03

- FAA40 is selectivity at age and fleet x F40:

1	2	3	4	5	6	7	8+
0.01	0.04	0.09	0.17	0.17	0.17	0.17	0.17
0.01	0.09	0.10	0.15	0.26	0.39	0.54	0.54
0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
0.03	0.08	0.15	0.21	0.24	0.25	0.25	0.25

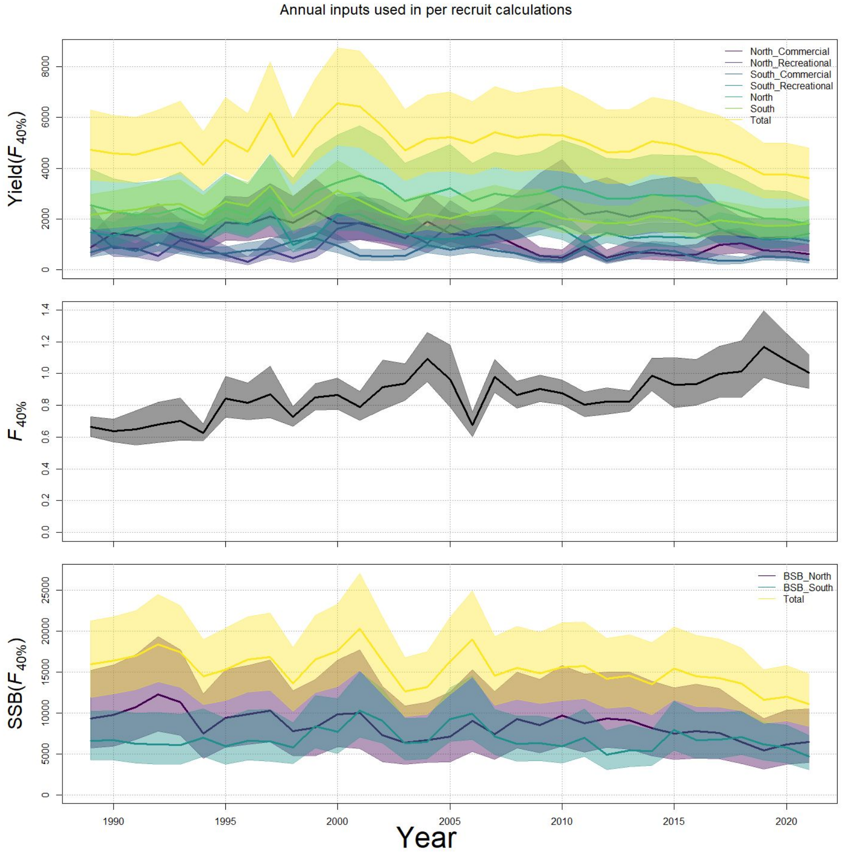
BRPs example

- Calculates joint uncertainty of status of current F and SSB relative to reference points:



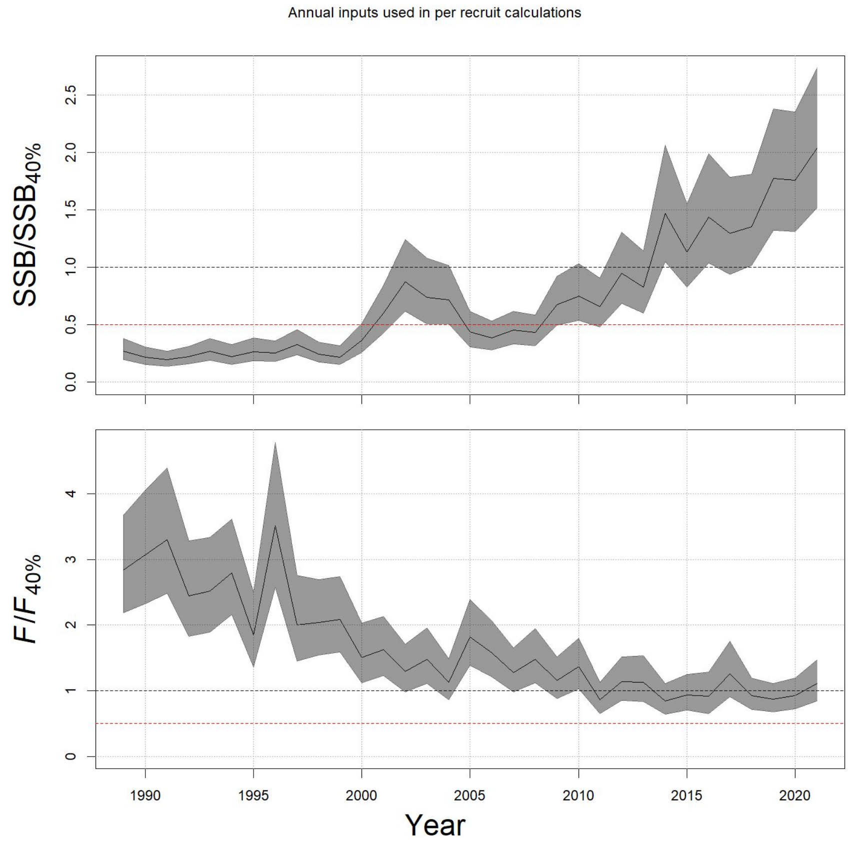
BRPs example

- Calculates reference points every year using annual inputs instead of last 5 years:



BRPs example

- Calculates annual status using annual reference points:



Projections

Multi-WHAM has the same options as the standard WHAM package

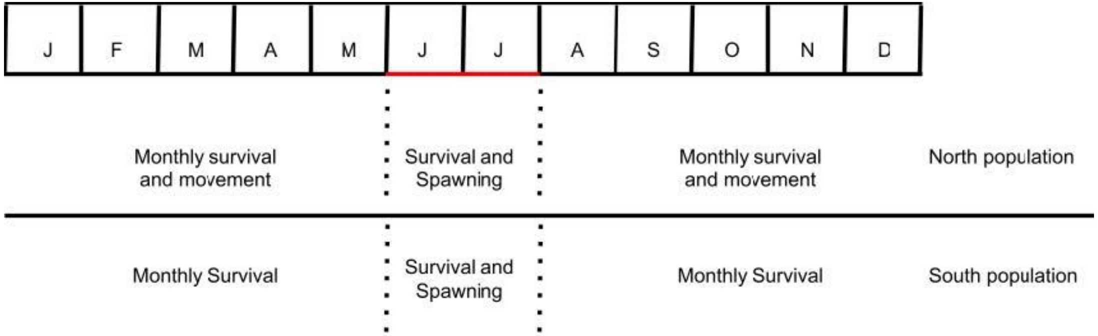
- Continues time series models for some random effects:
  - Recruitment
  - Survival/movement transitions
  - Environmental covariates
  - Optional for M, and movement
  - Uncertainty in these estimates grows in projection years moving away from observations
  - Under AR1 model projected random effects converge to the mean of the process
- For other dynamics user specifies years to average (default is the same as that for prevailing BRPs).
- Various options for projected (year-specific) fully-selected F or FAA:
  - Status quo
  - F40 (or some other percentage)
  - Fmsy (if a S-R function is used)
  - user-specified fully-selected F
  - user-specified total Catch (appropriate FAA is calculated internally)

Configuration for BSB

- 2 regions:
  - North
  - South
- 2 stock components:
  - North
  - South
- Model years: 1989 - 2021
- Ages: 1-8+
- Environmental covariate:
  - Bottom temperature in North (1959-2022)
- Natural mortality = 0.4 all ages, components, regions

Configuration for BSB

- All Jan 1 recruitment for a given stock component only in respective regions
- North fish can only move from south to north in first 5 intervals
- North fish can only move from north to south in last 4 intervals
- Any remaining North fish that are in the south move back to their spawning region at end of interval 5
- All North fish remain in North spawning region until end of interval 7
- Spawning season is only time when whole North population is in spawning region
- South population stays in South.



Configuration for movement (Northern component)

- The movement matrix for each interval of year after spawning:

$$\mathbf{p}_1 = \begin{bmatrix} 1 - p_1 & p_1 \\ 0 & 1 \end{bmatrix}$$

- Each interval of year before spawning:

$$\mathbf{p}_2 = \begin{bmatrix} 1 & 0 \\ p_2 & 1 - p_2 \end{bmatrix}$$

Movement rates from Stock Synthesis model

- The Stock Synthesis model has 2 intervals (6 months each)
- a proportion of the northern component moves to the south in one interval and some proportion move back to the south in the second interval.
- The movement matrices for each of the two intervals are:

$$\mathbf{P}_1 = \begin{bmatrix} 1 - P_1 & P_1 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{P}_2 = \begin{bmatrix} 1 & 0 \\ P_2 & 1 - P_2 \end{bmatrix}$$

Transforming between SS and WHAM

- Approximate the WHAM movement matrices as the roots of the SS matrices
  - roots defined by the number of WHAM intervals for each SS interval (5 and 4, respectively):
- Given the proportion parameter, the eigen decomposition of the matrices can be used to define the roots

$$\mathbf{P}_1^{\frac{1}{5}} = \mathbf{V}_1 \mathbf{D}_1^{\frac{1}{5}} \mathbf{V}_1^{-1}$$

$$\mathbf{P}_2^{\frac{1}{4}} = \mathbf{V}_2 \mathbf{D}_2^{\frac{1}{4}} \mathbf{V}_2^{-1}$$

Parameterizing the prior distributions

- The actual SS parameter estimates x_1 = -1.44 and x_2 = 1.94 are transformations of P_1 and P_2 such that

$$P_i = \frac{1}{1 + 2e^{-x_i}}$$

- Multi-WHAM uses an additive logit transformation

$$p_i = \frac{1}{1 + e^{-y_i}}$$

Parameterizing the prior distributions

Used a parametric bootstrap approach:

- Simulate 1000 values from a normal distribution with mean and standard deviation defined by the SS parameter estimate and standard error
- For each simulated value
  - construct $\mathbf{P}_i$
  - take the appropriate root,
  - calculated inverse logit for y_i
- calculate the mean and SD of the simulated values
- mean values over simulations did not differ meaningfully from the transformation of the original estimates:
- SD was approximately 0.2 for both parameters.
- distributions for random effects defining the movement parameters configured using the mean and SD from the bootstraps.

Initial abundance at age

- With the movement configuration, northern origin fish (ages 2+) can occur in the southern region on January 1.
- Estimating initial numbers at age as separate parameters can be challenging even in single-stock models.
- To avoid difficulties, we used the equilibrium assumption described previously.
- Two parameters are estimated for each regional stock component: an initial recruitment and an equilibrium full F across all fleets.

Recruitment and survival/movement transitions

- 2DAR1 (age and year) correlated random effects for both the northern and southern components.
- Variance and correlation parameters are different for the northern and southern components.
- Northern component:
  - abundance at age 1 on January 1 (recruitment) is only allowed in the northern region,
  - older individuals may occur in either region on Jan 1 (based on movement description)
  - survival random effects will occur for abundances at age in both regions.
  - Base model assumes very small variance for the transitions in the southern region (approximately SCAA)
  - 2DAR1 models with estimated variance for southern region would not converge (correlation could not be estimated).

Observations

- Aggregate catch: 2 fleets in each region:
  - Commercial (1989-2021)
  - Recreational (1989-2021)
- Aggregate indices: 2 in each region:
  - Spring VAST (1989-2021)
  - Recreational CPA (1989-2021)
- Age composition for all fleets and indices (1989-2021)
- Model-based bottom temperature observation in Northern region (1959-2022)

Age-composition likelihoods

Data component	Age Composition Likelihood
North Commercial	Dirichlet-Multinomial
North Recreational	Logistic-normal (0s as missing)
South Commercial	Logistic-normal (AR1, 0s as missing)
South Recreational	Logistic-normal (AR1, 0s as missing)
North Recreational CPA	Logistic-normal (0s as missing)
North VAST	Dirichlet-Multinomial
South Recreational CPA	Logistic-normal (AR1, 0s as missing)
South VAST	Logistic-normal (AR1, 0s as missing)

Selectivity

Data component	Mean Selectivity model	Random effects configuration
North Commercial	age-specific (flat-topped at ages > 3)	2D-AR1 (age and year)
North Recreational	age-specific (flat-topped at ages > 6)	2D-AR1 (age and year)
South Commercial	logistic	None
South Recreational	logistic	None
North Recreational CPA	age-specific (flat-topped at ages > 1)	AR1 (year)
North VAST	age-specific (flat-topped at ages > 4)	2D-AR1 (age and year)
South Recreational CPA	age-specific (flat-topped at ages > 2)	None
South VAST	age-specific (flat-topped at ages > 1)	None

Uncertainty in Recreational CPA indices

- CVs provided by analyses that generated Rec CPA indices were deemed implausibly small (CVs: 0.02 to 0.06).
- We estimated a scalar of the SD of the log-aggregate Rec CPA indices.
- Estimates of the scalar were usually approximately 5 for the north and the south Rec CPA indices.
- Estimation included in the proposed base model allow more realistic estimates of uncertainty in model output.

Bottom Temperature effects on recruitment

- Included model-based bottom temperature observations in the BSB model.
- Very small uncertainty in observations (SEs: 0.03 to 0.09).
- State-space treatment:
  - Modeled latent covariate as AR1 process

$$X_y \sim N\left(\mu_X(1-\rho_X) + \rho_X X_{y-1}, \sigma_X^2\right)$$

  - Observations of the latent covariate:

$$x_y \sim N\left(X_y, \sigma_x^2\right)$$

- Effect of latent covariate on northern recruitment

$$\log R_y = \mu_R + \beta X_y + \epsilon_y.$$